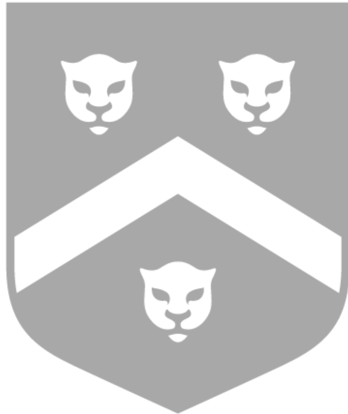

Graphs

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Graph ADT
Graphs
Terminology
Graph Class

Graph ADT

Graphs

- Trees describe relationships in a strict hierarchy
- Graphs describe networks with more interconnected relationships
- Graphs can represent many useful things:
  - Devices on an electronic network
  - Components on a silicon chip
  - Road maps
  - Course prerequisites
  - States and transitions in a system

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Terminology

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Definition

- A graph is a data structure that consists of a set of *vertices* (or nodes) and a set of *edges* (relations) between pairs of vertices
- Edges represent paths or connections between vertices
- Both the set of vertices and the set of edges must be finite
- Either set may be empty

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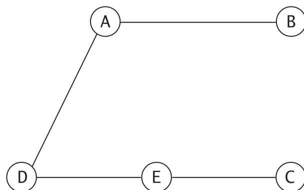
Example

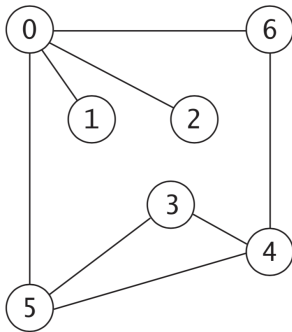
V = {A, B, C, D, E}
E = {{A, B}, {A, D}, {C, E}, {D, E}}

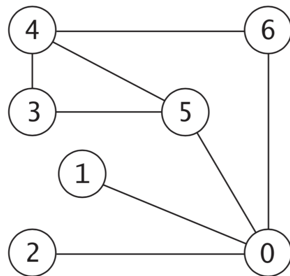


- Each edge is represented by the two vertices it connects
- If there is an edge between vertices x and y, there is a *path* from x to y and vice versa

Visualization

The physical layout of the vertices and their labeling is not relevant





Two equivalent graphs

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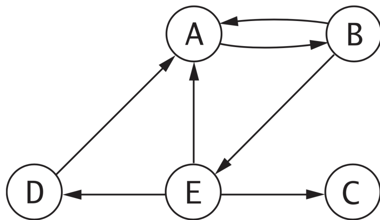
Edge Features

- Edges are *undirected* if they represent a transition in both directions
- Edges are *directed* if they represent a transition in only one direction
- Edges are *unweighted* if all transitions' costs are equal
- Edges are *weighted* if there are different costs associated with different transitions



Directed graph with arrowed edges

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More Terminology

- Two vertices are *adjacent* and *neighbors* if there is an edge from one vertex to the other
- A *path* is a sequence of edges between adjacent vertices
- A *simple path* is a sequence with all unique edges and vertices (except maybe the first/last vertex)
- A *cycle* is a simple path with the same start and end vertex

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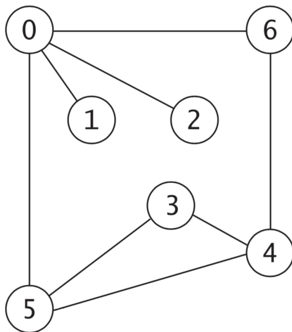
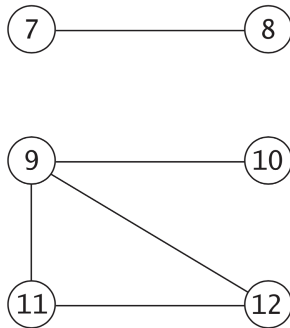
Edge Features

- A graph is *connected* if there is a path from every vertex to every other vertex
- A *connected component* is a subset of vertices that are connected





An *unconnected* graph with three connected components

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Requirements

- Java does not provide a Graph data structure
- Desired operations:
  - Create a graph with a specific number of vertices
  - Iterate through all vertices
  - Iterate through all neighbors of a vertex
  - Insert an edge
  - Iterate through all edges
  - Check if an edge exists
  - Determine edge weight

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Graph Class

Data Field	Attribute
private int dest	The destination vertex for an edge.
private int source	The source vertex for an edge.
private double weight	The weight.
Constructor	**Purpose**
public Edge(int source, int dest)	Constructs an Edge from source to dest. Sets the weight to 1.0.
public Edge(int source, int dest, double w)	Constructs an Edge from source to dest. Sets the weight to w.
Method	**Behavior**
public boolean equals(Object o)	Compares two edges for equality. Edges are equal if their source and destination vertices are the same. The weight is not considered.
public int getDest()	Returns the destination vertex.
public int getSource()	Returns the source vertex.
public double getWeight()	Returns the weight.
public int hashCode()	Returns the hash code for an edge. The hash code depends only on the source and destination.
public String toString()	Returns a string representation of the edge.

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Vertices and Edges

- Vertex representations:
  - Each vertex is represented by an integer, starting at 0
- Edge class:
  - Requires source vertex
  - Requires destination vertex
  - Requires weight
  - Edges are directed so we will use two edges to represent a single edge in an undirected graph

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Representations

- There are two common graph representations
- Depending on the intended use, one or the other is more efficient
- Adjacency list:
  - Uses an array of lists
  - Each element represents a vertex, and each entry in its list represents adjacent vertices
- Adjacency matrix:
  - Uses a square two-dimensional array
  - Each element records whether there is a connection from the vertex in that row to the vertex in that column
  - Elements can hold weights as well

Adjacency List

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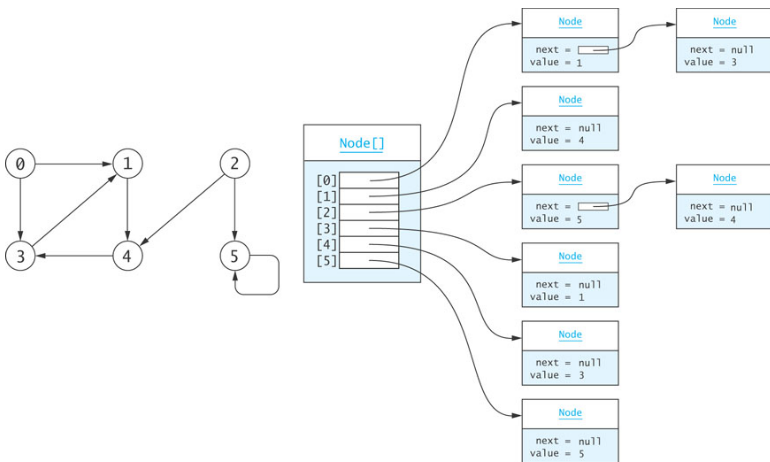

Array of linked lists that hold adjancent nodes

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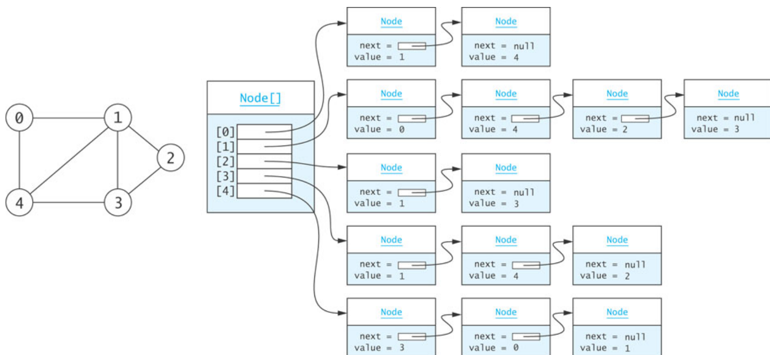
Adjacency List



Undirected graph with edges going both ways

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AbstractGraph Class

Data Field	Attribute
`private boolean directed`	**`true`** if this is a directed graph.
`private int numV`	The number of vertices.
Constructor	**Purpose**
`public AbstractGraph(int numV, boolean directed)`	Constructs an empty graph with the specified number of vertices and with the specified `directed` flag. If `directed` is **`true`**, this is a directed graph.
Method	**Behavior**
`public int getNumV()`	Gets the number of vertices.
`public boolean isDirected()`	Returns **`true`** if the graph is a directed graph.
`public void loadEdgesFromFile(Scanner scan)`	Loads edges from a data file.
`public static Graph createGraph (Scanner scan, boolean isDirected, String type)`	Factory method to create a graph and load the data from an input file.

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ListGraph Class

Data Field	Attribute
`private List<Edge>[] edges`	An array of `List`s to contain the edges that originate with each vertex.
Constructor	**Purpose**
`public ListGraph(int numV, boolean directed)`	Constructs a graph with the specified number of vertices and directionality.
Method	**Behavior**
`public Iterator<Edge> edgeIterator(int source)`	Returns an iterator to the edges that originate from a given vertex.
`public Edge getEdge(int source, int dest)`	Gets the edge between two vertices.
`public void insert(Edge e)`	Inserts a new edge into the graph.
`public boolean isEdge(int source, int dest)`	Determines whether an edge exists from vertex `source` to `dest`.

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Data fields

```
import java.util.*;

/** A ListGraph is an extension of the
    AbstractGraph abstract class that uses an array
    of lists to represent the edges. */
public class ListGraph extends AbstractGraph {
  // Data Field
  /** An array of Lists to contain the edges that
      originate with each vertex. */
  private List<Edge>[] edges;
. . .
}
```

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Constructor

```
/** Construct a graph with the specified number of
    vertices and directionality.
    @param numV The number of vertices
    @param directed The directionality flag
*/
public ListGraph(int numV, boolean directed) {
  super(numV, directed);
  edges = new List[numV];
  for (int i = 0; i < numV; i++) {
    edges[i] = new LinkedList<Edge>();
  }
}
```

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isEdge

```
/** Determine whether an edge exists.
    @param source The source vertex
    @param dest The destination vertex
    @return true if there is a (src, dst) edge
*/
public boolean isEdge(int src, int dst) {
  return edges[src].contains(new Edge(src, dst));
}
```

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insert and edgeIterator

```
/** Insert a new edge into the graph.
    @param edge The new edge
*/
public void insert(Edge edge) {
  edges[edge.getSource()].add(edge);
  if (!isDirected()) {
  edges[edge.getDest()].add(new Edge(edge.getDest(),
                              edge.getSource(),
                              edge.getWeight()));
  }
}
```

```
public Iterator<Edge> edgeIterator(int source) {
  return edges[source].iterator();
}
```

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getEdge

```
/** Get the edge between two vertices. If an
    edge does not exist, an Edge with a weight
    of Double.POSITIVE_INFINITY is returned.
    @param source The source
    @param dest The destination
    @return the edge between these two vertices
*/
public Edge getEdge(int source, int dest) {
  Edge target =
    new Edge(source, dest, Double.POSITIVE_INFINITY);
  for (Edge edge : edges[source]) {
    if (edge.equals(target))
      return edge; // Desired edge found, return it.
  }
  return target; // Desired edge not found.
}
```

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Graph Algorithms

- Graphs support a variety of analysis to solve many problems
- Many algorithms follow common forms, though the specifics vary

ANALYZEGRAPH(G)

1: **for** vertex u in G **do**
2:     **for** each vertex v adjacent to u **do**
3:         Do something with vertex v or edge (u,v)