Hash Tables

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Hash Table

Hash Table Description

- **Hash tables** are implementations of data storage with useful properties:
  - Used to implement sets
  - Used to implement maps
  - Hash tables store keys (and maybe values)
  - These keys (and associated values) are directly accessible
  - Similar to an index in an array, there's only one location an entry might be in a hash table

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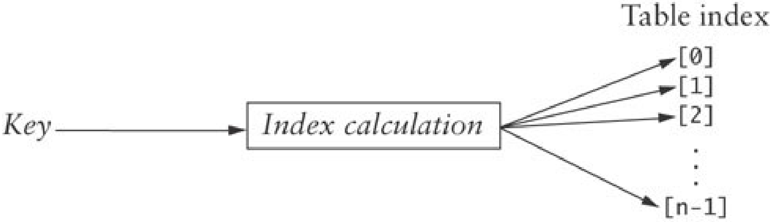
Definition

- A **Hash table** consists of two necessary parts:
  - An array to hold values – the table
  - A **hash function** which translates a key to an integer value called a **hash code**
  - The integer value is used as an array index – Java arrays only ever use `int`s as an index



Example Hash Function

- Consider an array of size 20, and characters for keys
- An example hash function could be: convert the character to ASCII, and then `mod` the result by 20
- With this function, all the resulting indices are between $0-19$, and each character shows up at a predictable location
- For example, $A = 65$, so its index is $65\%20 = 5$. $a = 97$, so its index is $97\%20 = 17$
- We can store/retrieve these characters by looking directly at their associated index, no searching needed
- Is there an issue with this?

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Hash Codes

- Usually keys are strings of letters/numbers
- The number of possible keys is much larger than the table
- Different keys can generate the same hash code, causing a *collision*
- A good hash function distributes all of the keys evenly across possible indices
- Researchers have written better hash functions already – we typically use those rather than create our own

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Java `HashCode` Method

- Java's string hash function is called with the `.HashCode()` method
- Both the individual characters and their position in the string have an effect on the hash code
- string s has the hash code $s_0 \times 31^{n-1} + s_1 \times 31^{n-2} + \cdots + s_{n-1}$
- Example: `"Cat".HashCode() =` $\text{‘C’} \times 31^2 + \text{‘a’} \times 31 + \text{‘t’} = 67510$
- 31 is chosen as a multiplier because it is prime, which gives good distribution properties usually

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Collisions

- `.HashCode()` distributes hash codes evenly, so one index isn't more likely, given a range of keys
- The probability of a collision is based on how full the table is
- There is **always** a non-zero chance of a collision
- We will look at two ways to handle collisions without losing information

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Open Addressing

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Open Addressing

- Open addressing can be used to find/add items to a hash table without collision issues
- If there is a collision inserting a key, use *linear probing* to find other possible spots for the key:
  - Increment the index by 1 until there is a null element
  - Store the key there
- If there is a collision searching for a key, follow the same steps:
  - Increment the index by 1 until the key is found or there is a null element

Issues

- What happens if you reach the end of the array?
  - Treat the array like a circular array
  - Set the index to 0 and then start incrementing again
- What happens if the array is full?
  - We will search for a null spot forever
  - Instead, detect an end condition: when we get back to our starting spot
- Avoid a full table by resizing after a certain *load factor*

Example

Key	"Tom"	"Harry"	"Sam"	"Pete"
`hashCode()`	84274	69496448	82879	2484038
`hashCode() % 5`	4	3	4	3

Inserting Tom:

				Tom

Example

Key	"Tom"	"Harry"	"Sam"	"Pete"
`hashCode()`	84274	69496448	82879	2484038
`hashCode() % 5`	4	3	4	3

Inserting Harry:

			Harry	Tom

Example

Key	"Tom"	"Harry"	"Sam"	"Pete"
`hashCode()`	84274	69496448	82879	2484038
`hashCode() % 5`	4	3	4	3

Inserting Sam:

Sam			Harry	Tom

Example

Key	"Tom"	"Harry"	"Sam"	"Pete"
`hashCode()`	84274	69496448	82879	2484038
`hashCode() % 5`	4	3	4	3

Inserting Pete:

Sam	Pete		Harry	Tom

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Deletion

- We can't just set an index to null to delete that item with open addressing
- What if there had been a collision?
  - Set index to a dummy node – space is available for insertion but you should continue searching for find operations
- The dummy node can be replaced with a new key if that key is not in the table

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Resizing

- More collisions means more steps for each insert/find/delete operation
- Move all elements to a larger table so there are fewer collisions:
  - Create a larger array (ideally with a prime number of elements)
  - Insert all of the elements in the current array into the new one
  - Note that this requires *rehashing* – the indices might change with a different table size
  - Do not copy dummy values

Resizing Example

Key	"Tom"	"Harry"	"Sam"	"Pete"
`hashCode()`	84274	69496448	82879	2484038
`hashCode() % 5`	4	3	4	3
`hashCode() % 11`	3	10	5	7

Sam	Pete		Harry	Tom

Reinsert keys at new indices:

			Tom		Sam		Pete		Harry	

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Probing

- Linear probing leads to clusters of values in adjacent indices, which is inefficient
- *Quadratic* probing changes the increments when there is a collision
- Use square increments: $+1^2, +2^2, +3^2 \ldots$, using a circular array
- This spreads out colliding keys
- Issue: This sequence doesn't reach every index
- Solution: If the array has a prime size and enough free space, it will succeed

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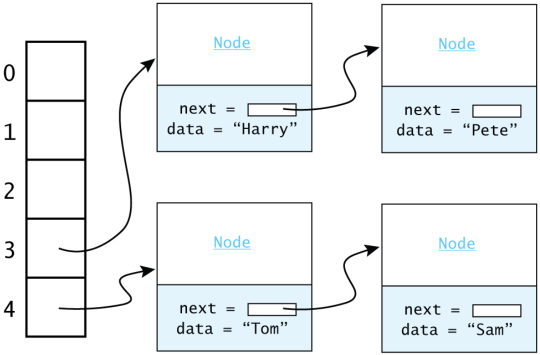
Chaining

Chaining Description

- *Chaining* is an alternative solution to hash code collisions
- Instead of each element in the table holding a value, each element holds a linked list
- These linked lists are called *buckets*



Advantages

- You only need to examine keys in a single bucket
- You can store more unique keys than the hash table size
- Insertion is simple – if the key is not in its bucket, insert it at the beginning of the list
- Deletion is simple – remove the key from the linked list

Performance

- For both open addressing and chaining hash tables, the load factor measures the number of non-null elements divided by table size
- The load factor determines how quickly we can insert/find/delete because it determines the chance of a collision
- Open addressing works more slowly than chaining when load factors are high
- Open addressing doesn't require linked lists so it is more memory-efficient
- When load factor is low, a hash table is as efficient as accessing values in an array, which is as efficient as possible

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put operation
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Data Fields

KWHashMap

KWHashMap Interface

Method	Behavior
`V get(Object key)`	Returns the value associated with the specified `key`. Returns **`null`** if the `key` is not present.
`boolean isEmpty()`	Returns **`true`** if this table contains no `key-value` mappings.
`V put(K key, V value)`	Associates the specified `value` with the specified `key`. Returns the previous `value` associated with the specified `key`, or **`null`** if there was no mapping for the `key`.
`V remove(Object key)`	Removes the mapping for this `key` from this table if it is present (optional operation). Returns the previous `value` associated with the specified `key`, or **`null`** if there was no mapping.
`int size()`	Returns the size of the table.

`Entry` Class

Data Field	Attribute
`private K key`	The key.
`private V value`	The value.
Constructor	**Behavior**
`public Entry(K key, V value)`	Constructs an `Entry` with the given values.
Method	**Behavior**
`public K getKey()`	Retrieves the key.
`public V getValue()`	Retrieves the value.
`public V setValue(V val)`	Sets the value.

Class to hold key-value pairs for entries in a hashtable

get operation

GET(key)

```
index ← key.hashCode() % table.length
if index is negative then
    index += table.length
if table[index] is null then
    return null
for all e in list at table[index] do
    if e.key matches key then
        return e.value
return null
```

put operation

```
PUT(key, value)
index ← key.hashCode() % table.length
if index is negative then
    index += table.length
if table[index] is null then
    table[index] ← new linked list
Search list for key
if key in table then
    set new value of entry
    return old value of entry
else
    Insert new key/value pair into list
    Increment numKeys
    return null
```

remove operation

REMOVE(key)

```
index ← key.hashCode() % table.length
if index is negative then
    index += table.length
if table[index] is null then
    return null
Search list for key
if key in table then
    Remove entry from list
    Decrement numKeys
    return value
return null
```

Data fields

```
import java.util.*;
public class HashtableChain<K, V>
             implements KWHashMap<K, V> {
  // Insert inner class Entry<K, V> here.
  /** The table */
  private LinkedList<Entry<K, V>>[] table;
  /** The number of keys */
  private int numKeys;
  /** The capacity */
  private static final int CAPACITY = 101;
  /** The maximum load factor */
  private static final double LOAD_THRESHOLD = 3.0;
  public HashtableChain() {
    table = new LinkedList[CAPACITY];
  }
  ...
}
```